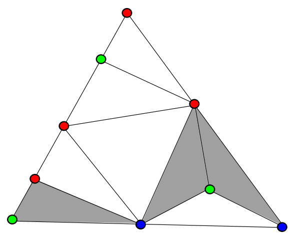
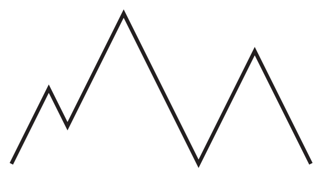
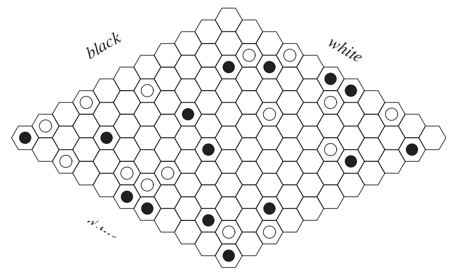
June 27 and 28, 2023. Starting with a handshake for G5, G7, G8.

Theorem 1 (The handshake lemma)**.** In every finite undirected graph, the number of vertices of odd degree is even.

Theorem 2 (Sperner's lemma)**.** If a big triangle is subdivided into smaller triangles meeting edge-to-edge, and the vertices are labeled with three colors so that only two of the colors are used along each edge of the big triangle (that is, a Sperner labeling), then at least one of the smaller triangles has vertices of all three colors.







Problem 1 (The mountain climbing problem)**.** For a piecewise linear function on a unit interval, with equal minimum values at the ends of the interval, it is possible to coordinate the motion of two points, starting from opposite ends of the interval, so that they meet somewhere in the middle while remaining at points of equal value throughout the motion.

Problem 2. The game of Hex is played by two players, who place pieces of their color on a tiling of a parallelogram-shaped board by hexagons until one player has a connected path of adjacent pieces from one side of the board to the other. Prove that the game can never end in a draw: by the time the board has been completely filled with pieces, one of the players will have formed a winning path.

Problem 3. Some integers in $\{1, 2, \dots, n\}$ are written on a line. Is it true that we can always divide the line into n parts and assign each part a label in $\{1, 2, \dots, n\}$ (each label is used exactly once) so that the part labeled by i contains at least as many copies of i's as any other part?

Problem 4 (Monsky's theorem)**.** It is not possible to dissect a square into an odd number of triangles of equal area.

Problem 5 (Karasev's dual theorem of the centerpoint theorem)**.** Given a family $\mathcal{F}$ $3n$ lines in $\mathbb{R}^2$, prove that there exists a point $x \in \mathbb{R}^2$ such that each ray from x intersects at least n lines in $\mathcal{F}$.

Problem 6 (Permutation generalization of Sperner's lemma)**.** Suppose that there are $n+1$ different Sperner's labelings $f_0, f_1, \dots, f_n$ of a triangulation of the n-dimensional simplex Δ_n. Prove that there exists a simplex in the triangulation whose vertices $v_0, \dots, v_n$ satisfy $f_i(v_i) = i$ for every $i \in \{0, 1, \dots, n\}$.